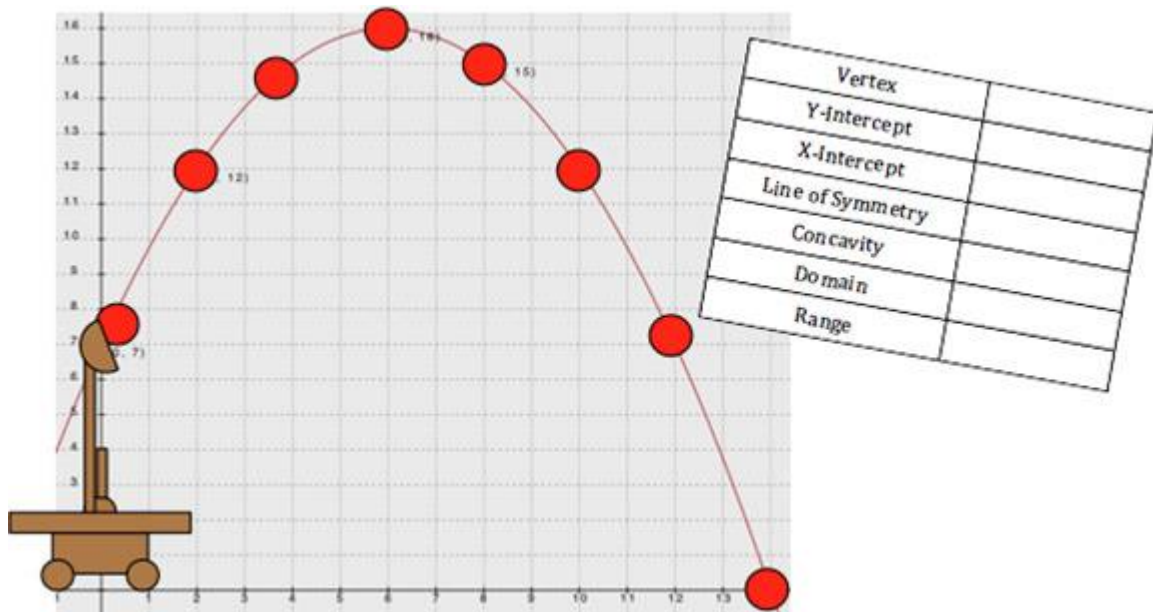


Quadratic Functions

Learning Quadratic Functions

- Completing square
- Drawing the graph

$$y = (x + p)^2 + q$$



Sketching quadratic functions

When sketching a quadratic function, we need to know:

1. roots (ie where the parabola cuts the x-axis)
2. the nature and coordinates of the turning point
3. y-intercept (ie where the parabola cuts the y-axis)

IMPORTANT

In this topic, we will also find out how to factorise a quadratic by 'completing the square'.

Example

Sketch the graph of $y = x^2 + 2x + 3$

Answer

First we need to complete the square to get the coordinates of the turning point.

$$y = x^2 + 2x + 3$$

$$y = (x + 1)^2 + 2$$

Therefore the coordinates of the turning point are (-1, 2).

If we recall the general equation: $y = ax^2 + bx + c$ then if:

- $a > 0$, then the shape of the parabola is like a happy face and the nature shows it is a minimum turning point
- $a < 0$, then the shape of the parabola is like a sad face and the nature shows it is a maximum turning point

Therefore, as $a > 0$ the above equation has a **minimum** turning point at (-1, 2).

Next, we need to find the roots of the equation. We can use the 'discriminant' to show how many roots there are, if any:

- $b^2 - 4ac > 0$ means there are two roots
- $b^2 - 4ac = 0$ means there is one root (because the turning point is on the x-axis)
- $b^2 - 4ac < 0$ means there are no roots

$$y = x^2 + 2x + 3$$

$$b^2 - 4ac \text{ where } a = 1, b = 2 \text{ and } c = 3$$

$$= 2^2 - (4 \times 1 \times 3)$$

$$= 4 - 12 = -8 \text{ which is } < 0 \text{ therefore there are no}$$

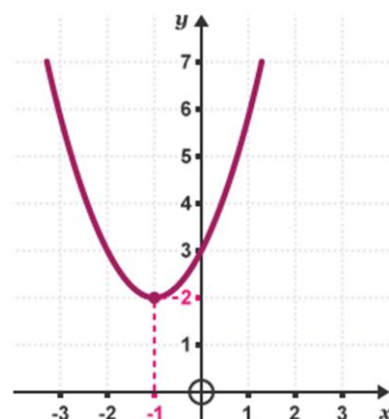
A parabola cuts the y axis, when $x = 0$:

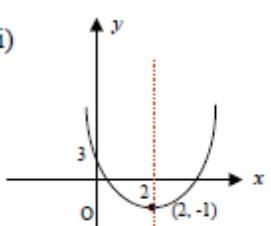
$$y = (0)^2 + 2(0) + 3$$

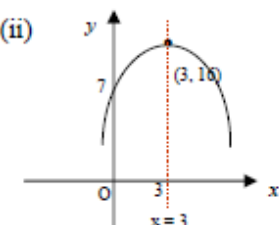
$$y = 3$$

Therefore the y-intercept is (0, 3)

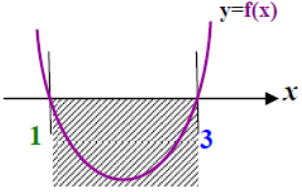
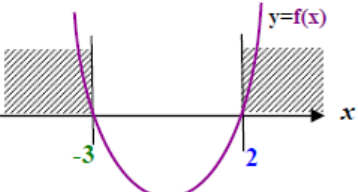
Therefore the graph looks like:



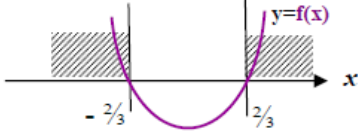
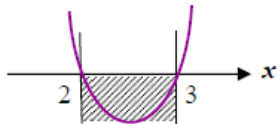
	EXAMPLE	EXERCISE
C1	<p>Express $f(x) = x^2 - 4x + 3$ in the form $(x + p)^2 + q$; with p and q as constants. Hence</p> <p>(i) State the minimum value of $f(x)$ and the corresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><u>Answers</u> : $a = 1 (> 0) \Rightarrow f$ has minimum value. $f(x) = x^2 - 4x + 3$</p> $= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 3$ $= (x-2)^2 - 4 + 3$ $= (x-2)^2 - 1$ <p>(i) Minimum value of $f(x) = -1$, when $x = 2$.</p> <p>(ii) </p> <p>Equation of axis of symmetry : $x = 2$.</p>	<p>L1. Express $f(x) = x^2 - 6x + 8$ in the form $(x + p)^2 + q$; with p and q as constants. Hence</p> <p>(i) State the minimum value of $f(x)$ and the corresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><u>Ans</u> :</p> <p>$p = -3, q = -1$</p>
L2	<p>Express $f(x) = x^2 + 2x - 3$ in the form $(x + p)^2 + q$. Hence</p> <p>(i) State the minimum value of $f(x)$ and the corresponding value of x.</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><u>Ans</u> :</p> <p>$p = 1, q = -4$</p>	<p>L3. Express $f(x) = x^2 + x + 2$ in the form $(x + p)^2 + q$. Hence</p> <p>(i) State the minimum value of $f(x)$ and the corresponding value of x.</p> <p>(iii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><u>Ans</u> :</p> <p>$p = \frac{1}{2}, q = \frac{7}{4}$</p>

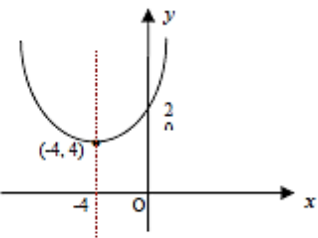
	EXAMPLE	EXERCISE
C2	<p>Express $f(x) = -x^2 + 6x + 7$ in the form $k - (x + p)^2$, k and p are constants. Hence</p> <p>(i) State the maximum value of $f(x)$ and state the coresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><i>Ans:</i> $a = -1 (< 0) \Rightarrow f$ has maximum value</p> $f(x) = -x^2 + 6x + 7$ $= -[x^2 - 6x - 7]$ $= -\left[x^2 - 6x + \left(\frac{-6}{3}\right)^2 - \left(\frac{-6}{2}\right)^2 - 7\right]$ $= -[(x - 3)^2 - 9 - 7]$ $= -[(x - 3)^2 - 16]$ $= 16 - (x - 3)^2$ <p>(i) Maximum $f(x) = 16$, when $x = 3$.</p> <p>(ii) </p> <p>Axis of symmetry is : $x = 3$.</p>	<p>L4. Express $f(x) = -x^2 - 8x + 9$ in the form $- (x + p)^2 + q$. Hence</p> <p>(i) State the maximum value of $f(x)$ and state the coresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><i>Ans:</i></p> <p>$p = 4, q = 25$</p>
L5	<p>Express $f(x) = -x^2 + 4x + 1$ in the form $-(x + p)^2$. Hence</p> <p>(i) State the maximum value of $f(x)$ and state the coresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><i>Jawapan :</i></p> <p><i>(sila gunakan kertas sendiri) $5 - (x - 2)^2$</i></p>	<p>L6. Express $f(x) = 4 - 3x - x^2$ in the form $q - (x + p)^2$ Hence</p> <p>(i) State the maximum value of $f(x)$ and state the coresponding value of x,</p> <p>(ii) Sketch the graph of $y = f(x)$ and state the equation of the axis of symmetry.</p> <p><i>Jawapan :</i></p> <p>$25/4 - (x + 3/2)^2$</p>

Inequalities

	EXAMPLE	EXERCISE
C1	<p>Solve $x^2 - 4x < -3$</p> <p>$x^2 - 4x + 3 < 0$ [In the form $f(x) < 0$] $(x - 1)(x - 3) < 0$ [faktorise]</p> <p>Consider $f(x) = (x - 1)(x - 3)$ $f(x) = 0 \Rightarrow x = 1$ atau $x = 3$</p>  <p>From the graph above, the range of x which satisfies the inequality $f(x) < 0$ ialah <u>$1 < x < 3$</u>.</p>	<p>L1. Solve $x^2 - 5x + 6 < 0$</p> <p>$2 < x < 3$</p>
C2	<p>Solve the inequality $x^2 + x - 6 \geq 0$</p> <p>$x^2 + x - 6 \geq 0$ $(x + 3)(x - 2) \geq 0$</p> <p>Consider $f(x) = 0$. Then $x = -3, x = 2$</p>  <p>Range of x is : $x \leq -3$ atau $x \geq 2$</p>	<p>L4. Solve the inequality $x^2 + 3x - 10 \geq 0$.</p> <p>$x \leq -5, x \geq 2$</p>
L5	<p>Solve the inequality $2x^2 + x > 6$.</p>	<p>L6. Solve the inequality $x(4 - x) \geq 0$.</p>

SPM

	EXAMPLE	EXERCISE
C1	<p>(≈ SPM 1998)</p> <p>(a) Given $f(x) = 9x^2 - 4$. Find the range of x for which $f(x)$ is positive.</p> <p>(b) Find the range of x which satisfy the inequality $(x - 2)^2 < x - 2$</p> <p><u>Ans</u> : (a) $f(x) > 0$ $9x^2 - 4 > 0$ $(3x + 2)(3x - 2) > 0$ $f(x) = 0 \Rightarrow x = -\frac{2}{3}, \frac{2}{3}$</p>  <p>$\therefore x < -\frac{2}{3}$ or $x > \frac{2}{3}$</p> <p>(b) $(x - 2)^2 < x - 2$ $x^2 - 4x + 4 - x + 2 < 0$ $x^2 - 5x + 6 < 0$ $(x - 2)(x - 3) < 0$</p>  <p>Range of x is $2 < x < 3$.</p>	<p>L1. (a) Given $f(x) = 2x^2 - 8$. Find the range of x so that $f(x)$ is positive.</p> <p>(b) Find the range of x which satisfy the inequality $(x - 1)^2 > x - 1$</p> <p>(Ans : (a) $x < -2, x > 2$ (b) $x < 1, x > 2$)</p>
L2	<p>(a) Find the range of x if $x(x + 2) \geq 15$</p> <p>(b) State the range of x if $5x > 2 - 3x^2$.</p> <p>(a) $x \leq -5, x \geq 3$ (b) $x < -2, x > 1/3$</p>	<p>L3. (a) Solve $2x(x - 3) < 0$</p> <p>(b) Find the values of x $x^2 > 4$.</p> <p>(a) $0 < x < 3$ (b) $x < -2, x > 2$</p>
L4	<p>(a) Find the range of x if $3x(2x + 3) \geq 4x + 1$</p> <p>(b) Solve $5 + m^2 > 9 - 3m$.</p> <p>(a) $x < -1, x > 1/6$ (b) $m < -4, m > 1$</p>	<p>L5. (a) Solve $-2x(x + 3) > 0$</p> <p>(b) Find the range of x if $9x^2 > 4$.</p> <p>(a) $-3 < x < 0$ (b) $x < -2/3, x > 2/3$</p>

	EXAMPLE /EXERCISE	EXERCISE
C2	<p>Given $f(x) = x^2 + 2kx + 5k$ (k constant) has a minimum value 4.</p> <p>(a) By completing the square, determine the TWO positive values of k</p> <p>(b) Sketch the graph of $y = f(x)$ for the bigger value of k and state the equation of the axis of symmetry.</p> <p><i>Answer:</i></p> <p>(a) $f(x) = x^2 + 2kx + 5k$</p> $= x^2 + 2kx + \left(\frac{2k}{2}\right)^2 - \left(\frac{2k}{2}\right)^2 + 5k$ $= (x+k)^2 - k^2 + 5k$ <p>$\therefore -k^2 + 5k = 4$ (minimum value)</p> $k^2 - 5k + 4 = 0$ $(k-1)(k-4) = 0$ $k = 1 \text{ or } k = 4$ <p>(b) $k = 4$, $f(x) = x^2 + 8x + 20$</p> $= x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 20$ $= (x+4)^2 - 16 + 20$ $= (x+4)^2 + 4$ <p>(ii)</p>  <p>Axis of symmetry : $x = -4$.</p>	<p>L6. Given $f(x) = x^2 + kx + 3$ (k constant) has a minimum value k.</p> <p>(a) By completing the square, determine the possible values of k</p> <p>(b) Sketch the graph of $y = f(x)$ for the value of k which is negative and state the equation of the axis of symmetry.</p> <p>(Ans: $k = -6$ atau 2)</p>