Quadratic Equation
General Equation

General equation for quadratic equation

$$y = ax^2 + bx + c$$

Example:
Express the quadratic equation in general form of $ax^2 + bx + c = 0$

$$4x(x + 4) = 10$$

Solution:
$4x(x + 4) = 10$
$4x^2 + 4x - 10 = 0$

Roots of Equation

What is roots?

Roots of an equation can be determine when $ax^2 + bx + c = 0$

	EXAMPLE	EXERCISE
C1.	Determine if -2 is a root of the equation $3x^2 + 2x - 7 = 0$. **x = -2, 3(-2)2 + 2(-2) – 7 = 12 – 4 – 7** **$\neq$ 0** **Hence - 2 is NOT a root of the given equation.**	L1. Determine if 3 is a root of the equation $2x^2 – x – 15 = 0$.
L2.	L1. **Determine if 3 is a root of the equation $x^2 – 2x + 3 = 0$.**	L3. **Determine if ½ is a root of the equation $4x^2 + 2x – 2 = 0$.**

Example:
Determine whether the $x = -3$ is the root for the equation $y = x^2 + 6x + 9$.

Solution:
Substitute x = -3 into the equation

$$y = (-3)^2 + 6(-3) + 9 = 0$$

Therefore, $x = -3$ is the root of quadratic equation.

Example:
By using substitution inspection method, determine -3 and 3 are roots of the quadratic equation
$x^2 - 9 = 0$

Solution:
$(-3)^2 - 9 = 0$
$(3)^2 - 9 = 0$

Finding Roots of Equation

Finding the roots of equation
a) Factorization
b) Completing Square
c) Using Formula

Example: Find the roots of the equation $y = x^2 + 8x + 15$ Solution Factorization $x^2 + 8x + 15 = 0$ $(x+5)(x+3) = 0$ X + 5 = 0 X = -5 Or X+3 = 0 X = -3 $x = -3 \; or \; x = -5$	Example: Find the roots of the equation $y = x^2 + 8x + 15$ Solution Using Completing Square $x^2 + 8x + 15 = 0$ $x^2 + 8x + 4^2 - 4^2 = -15$ $(x+4)^2 - 16 = -15$ $(x+4)^2 = 16 - 15$ $(x+4)^2 = 1$ $x + 4 = \pm\sqrt{1}$ $x + 4 = \pm 1$ $x + 4 = 1$ $x = -3$ or $x + 4 = -1$ $x = -5$	Example: Find the roots of the equation $y = x^2 + 8x + 15$ Solution Using Equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$ $x = -3 \; or \; x = \; -5$

Completing the square

***Example 4**: Solve the following equation by using completing the square method.*

a) $x^2+6x+5=0$

$x^2+6x=-5$ → Arrange in the form of $ax^2+bx=c$, where a = 1

$x^2+6x+\left(\frac{6}{2}\right)^2=-5+\left(\frac{6}{2}\right)^2$ → Add $\left(\frac{b}{2}\right)^2$ to both sides

$x^2+6x+(3^2)=-5+3^2$

$(x+3)^2=-5+9$

$(x+3)^2=4$

$x+3=\pm\sqrt{4}$ → Square root of a number you will get both positive & negative value

$x+3=2$ or $x+3=-2$

$\underline{x=-1}$ or $\underline{x=-5}$

b) $3y^2-5y+2=0$

$y^2-\frac{5}{3}y+\frac{2}{3}=0$ → You MUST reduce the coefficient of y^2 to 1, in this case divided the equation with 3.

$y^2-\frac{5}{3}y=-\frac{2}{3}$

$y^2-\frac{5}{3}y+\left(-\frac{5}{3}\text{x}\frac{1}{2}\right)^2=-\frac{2}{3}+\left(-\frac{5}{3}\text{x}\frac{1}{2}\right)^2$ → Add $\left(\frac{b}{2}\right)^2$ to both sides

$y^2-\frac{5}{3}y+\left(-\frac{5}{6}\right)^2=-\frac{2}{3}+\left(-\frac{5}{6}\right)^2$

$\left(y-\frac{5}{6}\right)^2=\frac{1}{36}$

$y-\frac{5}{6}=\pm\sqrt{\frac{1}{36}}$

$y-\frac{5}{6}=\frac{1}{6}$ or $y-\frac{5}{6}=-\frac{1}{6}$

$y=\frac{1}{6}+\frac{5}{6}$ $\qquad$ $y=\frac{5}{6}-\frac{1}{6}$

$\underline{y=1}$ $\qquad$ $\underline{y=\frac{2}{3}}$

Questions

L3.	Solve $(x-3)^2 = 1$. Ans : 2, 4	L4. Solve $1 + 2x^2 = 5x + 4$. Ans : 1, 3/2
L3.	Solve $(2x-1)^2 = 2x - 1$. Ans : ½ , 1	L4. Solve $5x^2 - 45 = 0$. Ans : – 3 , 3
L5.	Selesaikan $(x-3)(x+3) = 16$. Ans : – 5 , 5	L6. Selesaikan $3 + x - 4x^2 = 0$. Ans : – ¾ , 1

	EXAMPLE	EXERCISE
C3.	Solve $x^2 - 3x - 2 = 0$ by method of 'completing the square'. Give your answer correct to 4 significant figures. $x^2 - 3x - 3 = 0$ $x^2 - 3x + \left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2 - 2 = 0$ $\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 2 = 0$ $\left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$ $x - \frac{3}{2} = \pm\sqrt{\frac{17}{4}}$ $x = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$ $x = -0.5616$ atau $x = 3.562$	L5. Solve $x^2 + 5x - 4 = 0$. Give your answer correct to 4 significant figures. (Ans : 0.7016, -5.702)
L6.	Solve $x^2 + x - 8 = 0$. Give your answer correct to 4 significant figures. (Ans : 2.372, -3.372)	L7. Solve $x^2 + 7x + 1 = 0$, Give your answer correct to 4 significant figures. (Ans : -0.1459, -6.8541)

Question:

Find the following roots by using formula

a) $y = x^2 + 4x + 4$
b) $y = x^2 - 2x - 15$
c) $y = x^2 + 5x + 4$
d) $y = x^2 + 10x + 16$

Forming New Equation

$$y = x^2 - (\propto + \beta)x + \alpha\beta$$

Sum of Roots = $(\propto + \beta)$

Products of Roots = $\alpha\beta$

Example:

The roots of the equation $2x^2 - 5x - 6 = 0$. Find the new quadratic equation whose roots are $\frac{1}{\propto}$ *and* $\frac{1}{\beta}$

Solution:

$$2x^2 - 5x - 6 = 0.$$

$$x^2 - \frac{5}{2}x - \frac{6}{2} = 0$$

$$x^2 - \left(\frac{5}{2}\right)x + (-3) = 0$$

Sum of Roots   Product of Roots

$$Sum\ of\ Roots = \propto + \beta = \frac{5}{2}$$

$$Product\ of\ Roots = \propto \beta = -3$$

$$New\ sum\ of\ roots = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5/2}{-3} = -\frac{5}{6}$$

$$Products\ of\ roots = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{2}{5} \times \frac{1}{-3} = -\frac{2}{15}$$

So the new equation will be

$$y = x^2 - (Sum\ of\ Roots)x + (Product\ of\ Roots)$$
$$y = x^2 - \left(-\frac{5}{6}\right)x - \frac{2}{15}$$
$$y = x^2 + \frac{5}{6}x - \frac{2}{15}$$

The roots of the equation $3x^2 - 4x - 6 = 0$. Find the new quadratic equation whose roots are

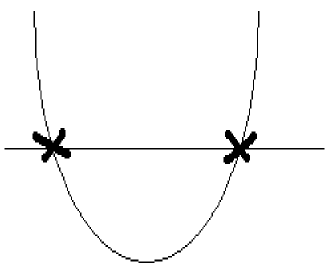
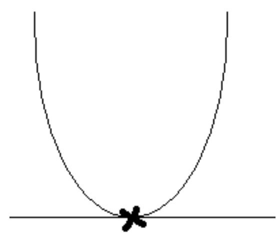
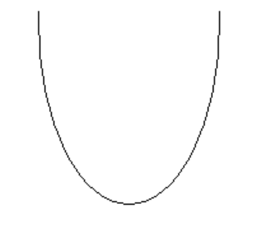
a) $2\alpha\ , 2\beta$

b) $\frac{1}{\propto}, \frac{1}{\beta}$

c) $\alpha + 1, \beta + 1$

Determine Types of Roots

Types of roots for quadratic functions

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
		

C1	**(SPM 2000)** **The roots of the quadratic equation $2x^2 + px + q = 0$ are - 6 and 3.** **Find** **(a) p and q,** **(b) range of values of k such that $2x^2 + px + q = k$ does not have real roots.** **Answer :** **(a)** $x = -6$, $x = 3$ $(x + 6)(x - 3) = 0$ $x^2 + 3x - 18 = 0$ $2x^2 + 6x - 36 = 0$ **Comparing :** $p = 6$, $q = -36$. **(b)** $2x^2 + 6x - 36 = k$ $2x^2 + 6x - 36 - k = 0$ $a = 2$, $b = 6$, $c = -36 - k$ $b^2 - 4ac < 0$ $6^2 - 4(2)(-36 - k) < 0$ $324 + 8k < 0$ $k < -40.5$	**L1. The roots of the quadratic equation $2x^2 + px + q = 0$ are 2 and -3.** **Find** **(a) p and q,** **(b) the range of values of k such that $2x^2 + px + q = k$ does not have real roots.**
L2	**Find the range of k if the quadratic equation $2x^2 - x = k$ has real and distinct roots.** (Ans : $k > -1/8$)	**L3. The quadratic equation $9 + 4x^2 = px$ has equal roots. Find the possible values of p.** (Ans : p = -12 atau 12)