

Integration

$$y = f(x) \xrightarrow{\text{differentiation}} \frac{dy}{dx} = g(x)$$

When you differentiate $f(x)$ you will get $g(x)$, however if you have integrate, then you would get back the $f(x)$

$$\int g(x) dx \xrightarrow{\text{integrate}} f(x)$$

First Case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

Example:

Integrate the following function $\int 2x dx$

Solution:

$$y = \int 2x dx = \frac{2x^2}{2} + c$$

<p>1. Integrate the following function</p> $\int 3x dx$ <p>Solution:</p>	<p>2. Integrate the following function</p> $\int 4x dx$ <p>Solution:</p>
<p>3. Integrate the following function</p> $\int 10x dx$ <p>Solution:</p>	<p>4. Integrate the following function</p> $\int 3x^2 dx$ <p>Solution:</p>

<p>5. Integrate the following function</p> $\int 3x^2 dx$ <p>Solution:</p>	<p>6. Integrate the following function</p> $\int 4x^2 dx$ <p>Solution:</p>
<p>7. Integrate the following function</p> $\int 4x^2 + 3x dx$ <p>Solution:</p>	<p>8. Integrate the following function</p> $\int 4x^4 dx$ <p>Solution:</p>
<p>9. Integrate the following function</p> $\int 4x^4 + 3x dx$ <p>Solution:</p>	<p>10. Integrate the following function</p> $\int 3x^4 + 2x^3 dx$ <p>Solution:</p>
<p>11. Integrate the following function</p> $\int 2x^4 + 3x^3 dx$ <p>Solution:</p>	<p>12. Integrate the following function</p> $\int 2x^4 + 3x^3 + x dx$ <p>Solution:</p>
<p>13. Integrate the following function</p> $\int 5x^5 + 2x dx$ <p>Solution:</p>	<p>14. Integrate the following function</p> $\int 8x dx$ <p>Solution:</p>

<p>15. Integrate the following function</p> $\int 3x^2 + 20 \, dx$ <p>Solution:</p>	<p>16. Integrate the following function</p> $\int 4x^2 + 12 \, dx$ <p>Solution:</p>
<p>17. Integrate the following function</p> $\int 4x^2 + 3x + 3 \, dx$ <p>Solution:</p>	<p>18. Integrate the following function</p> $\int 4x^4 + 5 \, dx$ <p>Solution:</p>
<p>19. Integrate the following function</p> $\int \frac{2}{\sqrt{x}} \, dx$ <p>Solution:</p>	<p>20. Integrate the following function</p> $\int \frac{24}{\sqrt{x}} \, dx$ <p>Solution:</p>
<p>21. Integrate the following function</p> $\int \frac{12}{\sqrt{x}} \, dx$ <p>Solution:</p>	<p>22. Integrate the following function</p> $\int -\frac{5}{\sqrt{x}} \, dx$ <p>Solution:</p>
<p>23. Integrate the following function</p> $\int -\frac{23}{\sqrt{x}} \, dx$ <p>Solution:</p>	<p>24. Integrate the following function</p> $\int \frac{4}{x^2} \, dx$ <p>Solution:</p>

<p>25. Integrate the following function</p> $\int \frac{-4}{x^2} dx$ <p>Solution:</p>	<p>26. Integrate the following function</p> $\int \frac{4}{x^4} dx$ <p>Solution:</p>
<p>27. Integrate the following function</p> $\int \frac{4}{x^5} dx$ <p>Solution:</p>	<p>28. Integrate the following function</p> $\int \frac{4}{3x^2} dx$ <p>Solution:</p>
<p>29. Integrate the following function</p> $\int \frac{6}{5x^2} dx$ <p>Solution:</p>	<p>30. Integrate the following function</p> $\int -\frac{4}{7\sqrt{x}} dx$ <p>Solution:</p>
<p>31. Integrate the following function</p> $\int \frac{12}{5x^4} dx$ <p>Solution:</p>	<p>32. Integrate the following function</p> $\int -\frac{12}{5x^4} dx$ <p>Solution:</p>
<p>33. Integrate the following function</p> $\int -\frac{12}{5x^{19}} dx$ <p>Solution:</p>	<p>34. Integrate the following function</p> $\int \frac{1}{5x^7} dx$ <p>Solution:</p>

<p>35. Integrate the following function</p> $\int (x + 3)(x + 5) dx$ <p>Solution:</p> $\int x^2 + 8x + 15 dx$ $= \frac{x^3}{3} + \frac{8x^2}{2} + 15x + c$ $= \frac{x^3}{3} + 4x^2 + 15x + c$	<p>36. Integrate the following function</p> $\int (x + 4)(x + 5) dx$ <p>Solution:</p>
<p>37. Integrate the following function</p> $\int (2x + 3)(x + 5) dx$ <p>Solution:</p>	<p>38. Integrate the following function</p> $\int (x + 3)(3x + 5) dx$ <p>Solution:</p>
<p>39. Integrate the following function</p> $\int (x^2 + 3)(x + 5) dx$ <p>Solution:</p>	<p>40. Integrate the following function</p> $\int (2x^2 + 3)(x + 5) dx$ <p>Solution:</p>
<p>41. Integrate the following function</p> $\int (x^2 + 5)(x + 4) dx$ <p>Solution:</p>	<p>42. Integrate the following function</p> $\int (x^5 + 3)(x + 5) dx$ <p>Solution:</p>
<p>43. Integrate the following function</p> $\int (x^4 + 3)(3x + 5) dx$ <p>Solution:</p>	<p>44. Integrate the following function</p> $\int (2x^6 + 3)(3x + 5) dx$ <p>Solution:</p>

Finding the value of C

Example:

Integrate the following function $\int 2x \, dx$.

Solution:

$$y = \int 2x \, dx = \frac{2x^2}{2} + c$$

<p>1. Integrate the following function and determine the C value given that $y= 5$ and $x= 2$</p> <p style="text-align: center;">$\int 3x \, dx$</p> <p>Solution:</p>	<p>2. Integrate the following function and determine the C value given that $y= 10$ and $x= 4$</p> <p style="text-align: center;">$\int 4x \, dx$</p> <p>Solution:</p>
<p>3. Integrate the following function and determine the C value given that $y= 8$ and $x= 8$</p> <p style="text-align: center;">$\int 10x \, dx$</p> <p>Solution:</p>	<p>4. Integrate the following function and determine the C value given that $y= 5$ and $x= 2$</p> <p style="text-align: center;">$\int 3x^2 \, dx$</p> <p>Solution:</p>

Determine the value of C given that $y = 15$ and $x = 3$

<p>5. Integrate the following function. Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 3x^2 dx$ <p>Solution:</p>	<p>6. Integrate the following function Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 4x^2 dx$ <p>Solution:</p>
<p>7. Integrate the following function Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 4x^2 + 3x dx$ <p>Solution:</p>	<p>8. Integrate the following function. Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 4x^4 dx$ <p>Solution:</p>
<p>9. Integrate the following function. Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 4x^4 + 3x dx$ <p>Solution:</p>	<p>10. Integrate the following function Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 3x^4 + 2x^3 dx$ <p>Solution:</p>
<p>11. Integrate the following function Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 2x^4 + 3x^3 dx$ <p>Solution:</p>	<p>12. Integrate the following function Determine the value of C given that $y = 15$ and $x = 3$</p> $\int 2x^4 + 3x^3 + x dx$ <p>Solution:</p>

Determine the equation of y given the differentiation below

<p>1. Given that $\frac{dy}{dx} = 2x + 5$, find the equation for y when y = 50 when x = 5</p>	<p>Given that $\frac{dy}{dx} = 2x^4 + 5$, find the equation for y when y = 30 when x = 4</p>
<p>Given that $\frac{dy}{dx} = 2x + \frac{5}{x}$, find the equation for y when y = 60 when x = 12</p>	<p>Given that $\frac{dy}{dx} = x^3 + 5$, find the equation for y when y = 16 when x = 4</p>
<p>Given that $\frac{dy}{dx} = 2x + 15$, find the equation for y when y = 10 when x = 1</p>	<p>Given that $\frac{dy}{dx} = 5x^3 + 5$, find the equation for y when y = 16 when x = 3</p>
<p>Given that $\frac{dy}{dx} = 8x + 5$, find the equation for y when y = 19 when x = 6</p>	<p>Given that $\frac{dy}{dx} = 2x^3 + x^2 + 5$, find the equation for y when y = 25 when x = 4</p>

Complex Integration

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

1. Given that $\int (2x + 4)^2$ find the equation for y when y = 50 when x= 5	Given that $\int (5x + 4)^4$ find the equation for y when y = 30 when x= 4
Given that $\int (12x + 4)^2$ find the equation for y when y = 60 when x= 12	Given that $\frac{dy}{dx} = (3x + 7)^4$, find the equation for y when y = 16 when x= 4
Given that $\int (3x + 4)^4$, find the equation for y when y = 10 when x= 1	Given that $\int (5x + 4)^3$, find the equation for y when y = 16 when x= 3
Given that $\int (7x + 4)^2$, find the equation for y when y = 19 when x= 6	Given that $\frac{dy}{dx} = (4x + 5)^3$, find the equation for y when y = 25 when x= 4

Forming Equation from Gradient

<p>1. Given that gradient of quadratic equation is $2x + 5$,</p> <p>a. find the equation for y when $y = 50$ when $x = 5$</p> <p>b. Determine the value of y when $x = 2$</p>	<p>2. Given that gradient of quadratic equation is $2x^4 + 5$</p> <p>a. find the equation for y when $y = 30$ when $x = 4$</p> <p>b. Determine the value of y when $x = 3$</p>
<p>3. Given that gradient of quadratic equation is $2x$,</p> <p>a. find the equation for y when $y = 60$ when $x = 12$</p> <p>b. Determine the value of y when $x = 10$</p>	<p>4. Given that gradient of quadratic equation is $x^3 + 25$,</p> <p>a. find the equation for y when $y = 16$ when $x = 4$</p> <p>b. Determine the value of y when $x = 2$</p>
<p>5. Given that gradient of quadratic equation is $2x + 15$,</p> <p>a. If the equation passes through $(2,3)$</p> <p>b. Determine the the value of y when $x = 5$</p>	<p>6. Given that gradient of quadratic equation is $2x^2 + 15$,</p> <p>c. If the equation passes through $(5,3)$</p> <p>d. Determine the the value of y when $x = 7$</p>

Different varieties

Question 1:

Given that the value of $\int_0^1 kx + 5x \, dx = 9$ find the value of k ?

Question 2:

Given that the value of $\int_0^2 f(x) \, dx = 6$

a) $\int_0^2 3f(x) \, dx$

b) $\int_0^2 3f(x) + 1 \, dx$

c) $\int_0^2 3f(x) + x \, dx$

d) $\int_0^2 3f(x) + 3x^2 \, dx$

Question 3:

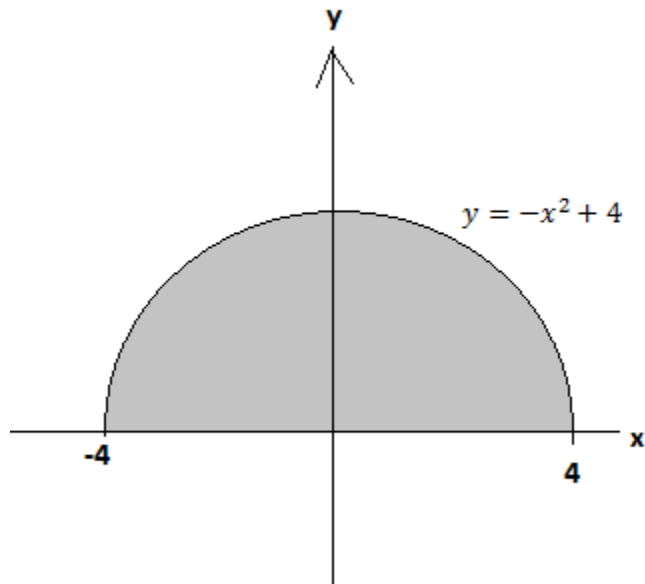
Given that $\int_0^6 3f(x) \, dx = 10$

a) $\int_0^2 3f(x) \, dx + \int_2^6 3f(x) \, dx$

b) $\int_0^6 4f(x) + 5x \, dx$

Integration

Use of integration to find the area of the shaded region



$area = \int_a^b y \, dx$:::: When the graph lying on the x axis

$area = \int_a^b x \, dy$:::: When the graph lying on the y axis

Note:

How to remember? Look at the diagram the equation is sleeping on the x - axis

Find the area of the shaded region.

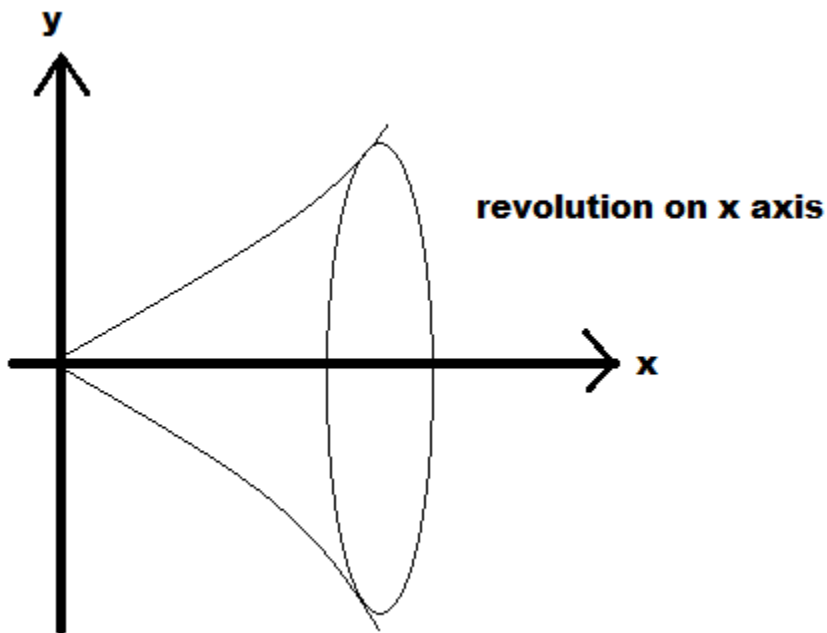
Solution:

$$Area = \int_{-4}^4 -x^2 + 4 \, dx$$

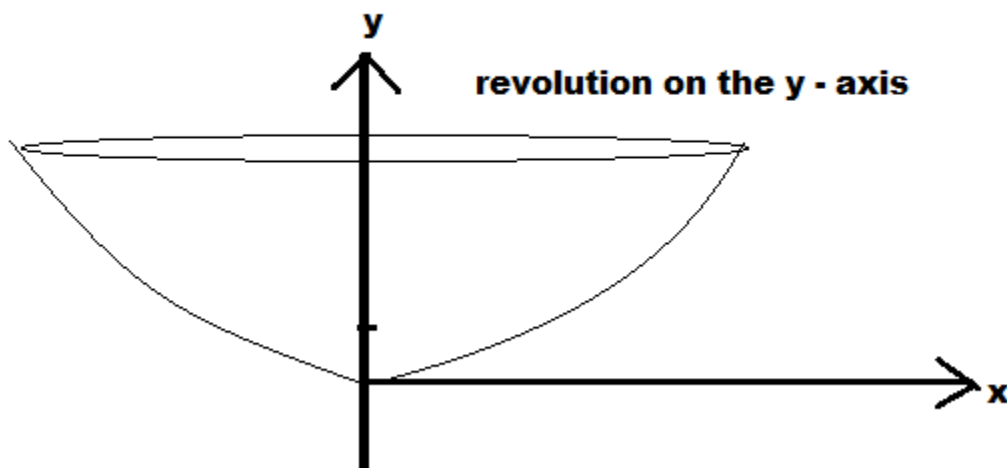
$$Area = \left[-\frac{x^3}{3} + 4x \right]_{-4}^4$$

.... Continue...

Volume



$Volume = \pi \int_a^b y^2 dx$:::: When the graph lying on the x axis

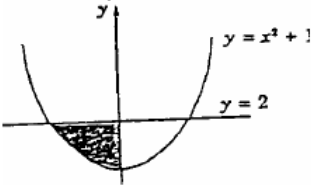
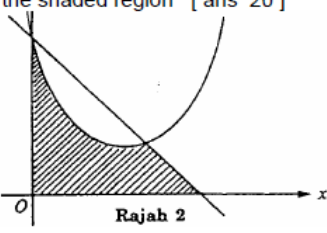
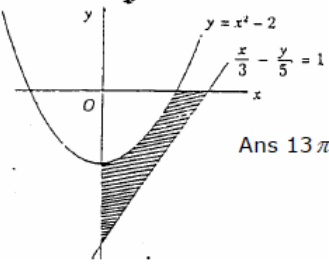


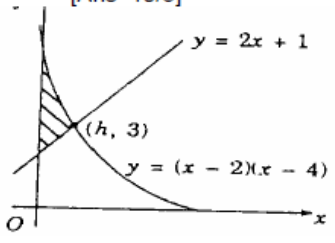
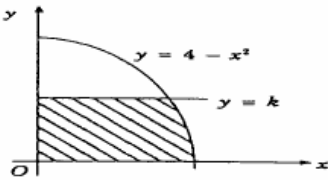
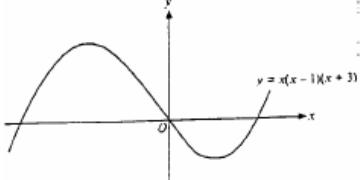
$Volume = \pi \int_a^b x^2 dy$:::: When the graph lying on the y axis

SPM QUESTIONS

<p>1. SPM 1993 a) Find i) $\int \frac{(4-x)(4+x)}{x^2} dx$</p>	<p>ii) $\int \frac{18}{(3x-5)^3} dx$</p>	<p>(b) Given that $\int_0^3 f(x) dx = 8$ find the value of $\int_0^3 \frac{f(x)+2}{2} dx$</p>
<p>2.SPM 1994 Find $\int (2x+7)^3 dx$</p>	<p>3. SPM 95 Evaluate a) $\int_1^2 \frac{2(x-3)(x+3)}{3x^2} dx$</p>	<p>b) Given that $y = \frac{2x-1}{x^2}$ and $\frac{dy}{dx} = 2g(x)$ where $g(x)$ is a function of x. Find the value $\int_{-1}^1 g(x) dx$</p>
<p>4. SPM 1996 a) Given $\int_0^4 f(x) dx = 5$ and $\int_1^3 g(x) dx = 6$ Find the value of a) $\int_0^4 2f(x) dx + \int_3^1 g(x) dx$</p>	<p>b) k if $\int_1^3 [g(x) - kx] dx = 14$</p>	<p>5. SPM 1997 If $\int_1^3 f(x) dx = 6$, find the value of $\int_1^3 [2f(x) - 5] dx$</p>
<p>6. SPM 98 (a) Find $\int x^2(2 + \frac{1}{x^4}) dx$</p>	<p>(b) Find the equation of a curve that has the gradient function $(2x+1)^3$ and passes through the point $(\frac{1}{2}, -3)$</p>	<p>7. SPM 1999 Given $\frac{d^2y}{dx^2} = 4x^3 + 1$. When $x = -1$ $y = \frac{1}{2}$ and $\frac{dy}{dx} = 3$. Express y in terms of x</p>

SPM Question

<p>SPM 93</p> <p>Calculate the volume of the solid generated when the shaded region in the diagram is revolved through 360° about the y-axis</p> <p>[Ans $\frac{1}{2}\pi$]</p> 	<p>SPM94</p> <p>Rajah 2 shows the graph of $y = x^2 - 4x + 7$ and $y = 7 - x$. Calculate the shaded region [ans 20]</p>  <p>Rajah 2</p>	<p>SPM98</p> <p>Diagram below shows the graph of $y = x^2 - 2$ and straight line $\frac{x}{3} - \frac{y}{5} = 1$</p> <p>Calculate the volume of the solid generated when the shaded region in the diagram is revolved through 360° about the y-axis</p>  <p>Ans 13π</p>
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<p>SM 97(a)</p> <p>The diagram below shows the line $y = 2x + 1$ and the curve $y = (x-2)(x-4)$ that intersect at point $(h, 3)$</p> <p>Find</p> <p>(i) the value of h [jb 1]</p> <p>(ii) area of the shaded region [Ans 10/3]</p> 	<p>SPM97(b)</p> <p>The diagram below shows the shaded region which is bounded by the curve $y = 4 - x^2$, $y = k$ and y-axis</p> <p>When the shaded region is revolved through 360° about y-axis, the volume generated is 6π unit³</p> <p>Find the value of k [Ans $k = 2$]</p> 	<p>SPM96</p> <p>The diagram below shows a curve $y = x(x-1)(x+3)$. Find the area which is bounded by the curve, line $x = -2$, $x = 1$ and x-axis.</p> <p>[Jb 9]</p> 
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